A FUZZY GROWTH OPTIONS APPROACH UNDER COMPETITION THREAT

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ABSTRACT

Information Communication Technology (ICT) projects may contain "wait and see" components, which give ICT managers the option to defer decisions until some uncertainty is resolved. In this paper, we treat these ICT opportunities as Real Options (RO) and assume that there is competition threat that can influence negatively or even worst eliminate their values. We consider exogenous competition modeling assuming that competitors are entering randomly into the market and cause a degradation of the available to the firm of interest overall market value. We introduce fuzzy logic and combine it with ROs under competition threat. The theory developed implicitly contains not only the deferral flexibility of projects but also the possibility of considering vague information, which needs to be taken into account when (long-time range) financial decisions are made. Our proposed model is applied to a real life broadband technology business activity associated with "Egnatia Odos S.A." strategic decision to deploy optical fiber backbone network along the national motorway "Egnatia Odos." The results of our model prove that FROs analysis may increase the overall value of the ICT business activity despite competition threat.

Key Words: real options, fuzzy logic, information communication technology, competition cost, net present value, investment analysis

INTRODUCTION

Information and Communication Technologies (ICT) lie at the convergence of information technology, telecommunications and data networking technologies. The valuation of ICT investments is a challenging task because it is characterized by high-level uncertainty and rapidly changing business conditions. Traditional finance theory suggests

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that firms should use a Discounted Cash Flow (DCF) methodology to analyze capital allocation requests. However, this approach does not properly account the flexibility inherent in most ICT investment decisions. ROs analysis presents an alternative method since it takes into account the managerial flexibility of responding to a change or new situation in business conditions (Trigeorgis 1996). An option gives its holder the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset in the future. Financial options are options on financial assets (e.g. an option to buy 100 shares of Nokia at 90€ per share on January 2007). Real Option (RO) is the extension of the options concept to real assets. For example, an ICT investment can be viewed as an option to exchange the cost of the specific investment for the benefits resulting from this investment. By adopting the philosophy of managerial flexibility (also called active management) we decrease the possibility of experiencing losses while increase the possibility of gaining. This is achieved by waiting and learning about the changing business conditions and generally resolving over time part of the overall investment's uncertainty (Trigeorgis 1996). For a general overview of real options, Trigeorgis (1996) provides an in-depth review and examples on different real options. For more practical issues the reader is referred to Mun (2002). Finally, Angelou and Economides (2005) provide a literature review of the ROs applications to real life ICT investments analysis.

After the liberalization of the telecommunications markets their market structure has changed from monopoly to oligopoly or perfect competition where many market participants are present. The real life ICT business activities do not belong exclusively to only one firm but may also be shared by other competitors. Viewing ICT projects as ROs, this paper develops a methodology for evaluating ICT investments decisions in the joint presence of uncertainty and competition. Our target is to develop a RO model closely related to the ICT industry characteristics to support ICT evaluation under competition conditions. As the number of players is increasing the exogenous competition modeling should take place since market conditions converge to perfect competition. In this case, a competitor's entry into the market will only cause a degradation of the overall ICT investment opportunity "pie" while the rest of the competitors will not react to this entry by changing their business strategy. On the other hand, in oligopolistic markets, actions taken by the firm may result to strategic reactions by its competitors. In this case, competition should be modeled endogenously requiring the combination of ROs and Game Theory (Zhu 1999).

In case of exogenous competition modeling the firm has to weight the value of waiting against the possible erosion of the value by competitor's actions, which it cannot influence. The firm has to determine what information has available about competition. If for example the firm knows in advance the strategies of its competitors and their impact on the firm's value function, the situation is completely deterministic. However, this case is quite unrealistic. In reality, the firm might have a rough idea about the intensity of competition and its impact without having full information about when and how other firms act (Trigeorgis 1996, Kumar 1999).

We model the cost of the competition as the part of the overall investment revenue V or alternatively "overall market pie," which is subtracted by the competitors, from the firm of interest. However, the uncertainty about competition may be genuine, meaning that we simply do not know the exact level of competition cost. In addition, while V is stochastic in nature according to ROs analysis we may also do not know its present level. To handle this, we introduce fuzzy logic and combine it with ROs under competition threat. The target is to find the optimum deployment strategy.

Kumar (1999) and Trigeogis (1996) model competition assuming that competitors are entering into the market following Poisson distribution. We also consider that the competitors are entering the market randomly according to an exogenous Poisson distribution. We relax existing literature assumptions by:

(i)   modelling competition costs I_{cwte}, I_{co} during waiting and operation period instead of the competitors arrival rate and competition erosion during these periods (Angelou and Economides 2006a, b).

(ii)   considering that the competition costs I_{cwte}, I_{co} are following a joint-diffusion processes with V and one time investment cost X.

(iii)   considering that the expected values for competition costs I_{cwte}, I_{co} and V and X are modeled by fuzzy logic.

A good example of many players in an ICT market, which is dominated by a strong player, is the Greek telecommunication market, which is dominated by the incumbent fixed telephony operator OTE (Hellenic Telecommunications Organization) (Kantor 2005, ITI 2005). After liberalization of the Greek market in 2001, an increasing number of new players has entered the market and started competing with the incumbent OTE in the

value-added services. However, none of them pose a significant threat to OTE. Actually, there are about 12 more players who possess low market shares compared to OTE. However, each of them may subtract some value from the overall business value of any new investment opportunity from OTE if the latter remains "inactive." For any new value added service, there is a market "pie" concerning its business activity that is usually growing over time. Some parts, of the whole "pie" will be subtracted by the competitors as they are entering in the market. So, the incumbent here faces a tradeoff between the value of flexibility to wait and the value of the possible competitive erosion during waiting period. The OTE's management has to determine whether it should exercise the option and implement the investment opportunity early or whether it should follow "wait-and-see" (WaS) strategy despite a competitive damage caused by the competitors' entry in the market.

The rest of the paper is organized as follows. In Section 2, we briefly present the ROs and fuzzy logic concepts and discuss the need for their integration. In Section 3, we provide a fuzzy ROs model under exogenous competition modeling. In addition, we specify our analysis in the ICT market mapping its characteristics to the competition parameters of our model. In Section 4, we put our analysis in the context of a specific real life case study. In Section 5, we discuss the relationship of method with other methods in the ICT investment analysis field. In addition, we provide limitations of our model and suggest possible future research. Finally, in Section 6, we conclude.

REAL OPTIONS AND FUZZY LOGIC INTEGRATION
Real Options

Spending money to exploit a business opportunity is analogous to exercising an option on, for example, a share of stock. It gives the right to make an investment's expenditure and receive an investment's asset, the value of which fluctuates stochastically. The amount of money spent for investment corresponds to the option's exercise price (X). The present value of the project's asset (total gain of investment) corresponds to the stock price (V). The length of time the company can defer the investment decision without losing the opportunity corresponds to the option's time to expiration (T). The uncertainty about the future value of the project's cash flows (the risk of the project) corresponds to the standard deviation of returns on the stock (σ). In general, the stock (σ) corresponds to the

variation in the cost and revenues of the investment. Finally, the time value of money is given in both cases by the risk-free rate of return (rf). The project's value as calculated by the real option methodology is the same with the value calculated by the Net Present Value (NPV) methodology when a final decision on the project can no longer be deferred (expiration date of the option). Table 1 summarizes the parameters' correspondence between a call option and an investment project. The total value of a project that owns one or more options is given by Trigeorgis (1999):

Expanded (Strategic) NPV = Static (Passive) NPV + Value of Options from Active Management (1)

The flexibility value named as option premium is the difference between the NPV value of the project as estimated by the Static or Passive Net Present Value (PNPV) method and the Strategic or Expanded NPV (ENPV) value estimated by the Real Options method. The higher the level of uncertainty, the higher the option value because the flexibility allows for gains in the upside and minimizes the downside potential. Sometimes it is hard to give a precise estimate of the expected value of underlying asset and it may be convenient to let it take interval values. Moreover, it may be the case that not all the members of the interval have the same reliability, as central members are more possible then the ones near the borders. The imprecision we encounter when judging or estimating future ICT investment cash flows is not only stochastic in nature, since the uncertainty may be genuine, i.e. we simply do not know the exact levels of present value of the expected future cash flows. We model this vagueness for the aforementioned investment parameters by adopting fuzzy logic analysis.

We adopt the real option rule in a more realistic setting by considering that the present values of expected cash flows and expected costs are estimated by triangular fuzzy numbers. This is exactly the idea behind our model. We model the expected values for investment revenue V, one time investment cost and competition costs during WaS and Operation Periods by adopting fuzzy logic analysis. Without introducing fuzzy real option models it would not be possible to formulate this genuine uncertainty. The proposed

model that incorporates subjective judgments and statistical uncertainties may give investors a better understanding of the problem when making investment decisions.

Table 1. Parameters' analogy between a call option and an investment opportunity

Investment Opportunity	Variable	Call Option
Present value of a project's assets or Present Value of cash flows from investment	V	Stock price
The amount of money spent for the investment	X	Agreed exercise price of the option
Investment expenditure required to exercise the option (cost of converting the investment opportunity into the option's underlying asset, i.e., the operational project)		
Length of time where the investment's decision may be deferred	T	Option's time to expiration (i.e., the maximum length of the deferral period)
Time value of money	r_f	Risk-free rate of return
Variance (Riskiness) of the investment's project assets (Costs, Revenues)	σ^2	Variance of returns on stock

## A Fuzzy Approach to Real Option Valuation

Most of the decision making in the physical world takes place in a situation in the pertinent data and the sequences of possible actions are not precisely known. Therefore, it is more realistic to adopt fuzzy data to express such situations in decision-making problems. Among all the different types of fuzzy numbers, the choice of using triangular numbers is made for the sake of simplicity, since assuming more complicated shapes may increase the computational complexity without substantially affecting the significance of the results. A fuzzy triangular number A on R ($-\infty, \infty$), with a membership function μA(x) is formally defined as follows:

$$A \overset{\Delta}{=} \mu_A(x) = \begin{cases} \dfrac{x - a_l}{a_m - a_l} \text{ for } a_l \leq x \leq a_m, \\[2ex] \dfrac{x - a_h}{a_m - a_h} \text{ for } a_m \leq x \leq a_h, \\[3ex] 0 \text{ otherwise} \end{cases} \qquad (2)$$

Where $[a_l, a_h]$ is the range of values (interval of smallest and largest possible value) and the point $(a_m, 1)$, the most possible, is the peak (see Figure 1).

In case of am=$(a_l + a_m)/2$ we say that equations (2) represent a central triangular fuzzy number. Triangular, numbers are very often used in the application (fuzzy controllers, managerial decision making, business and finance, social sciences, etc.) (Bojadziev and Bojadziev 1997). They have a membership function consisting of two linear segments joined at the peak $(a_m, 1)$, which makes graphical representations and operations with triangular number very simple. Also, it is important that they can be constructed easily on the basis of little information.

Figure 1. Triangular Fuzzy Number
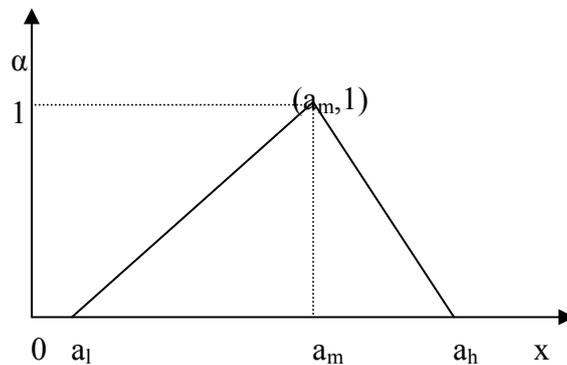
In this work we use triangular possibility distribution for the investment parameters used for the RO estimation. Usually, the present value of expected cash flows as well as one-time investment cost (option exercise cost) cannot be characterized by single numbers.

In addition, competition intensity in ICT market especially after its liberalization makes the estimation of the expected competition cost during waiting and operation phase a difficult task. In our analysis we consider that the expected values for the investment opportunity are in triangular ranges. We fix the peak value of the fuzzy numbers equal to the crisp value of the most expected value and we allow the nearby prices to have some degree of possibility. In our analysis we focus on a new ICT operator, called Firm, which is examining to enter to a new ICT market.

A FUZZY RO MODEL UNDER COMPETITION THREAT

We define T as the maximum deferral or "Wait-and-See" (WaS) period of the RO. During this period the option is shared among competitors. We assume that after this period no option exists at all for any competitor. The maximum deferral period is separated in two sub-periods, as seen in Figure 2. In the first sub-period, the Firm is not investing and is waiting for resolving some of the uncertainties associated with this investment opportunity. The second sub-period starts when the Firm exercises its option. For simplicity, we assume that the investment period (construction period for the specific project) is zero. The WaS period starts at t_s (assume $t_s=0$) when the option is available to the Firm. Also, t_e is the real exercise time of the option (implementation of the investment opportunity). Finally, the part of the operation period where the Firm can still face Competition Threat (CT) is $T-t_e$. All the notations used in our model are given in Table 2 in Appendix B. In addition, we define two terms for modeling the competition conditions: i) Preemption Threat from Competitors (PTC) and ii) Preemption Capability of Firm (PCF). PTC indicates the threat, which is experienced by the Firm during the WaS period of the option that other competitors may enter into the market and decrease or even more eliminate the option value. PCF indicates the capability of the Firm to preempt the subsequent competitors after its entry time at $t= t_e$ into the market.

During the WaS period, competitors may enter the market causing degradation of the investment opportunity for the Firm. We want to estimate the option value when there is a PTC against the Firm. The business target of the Firm is to minimize the threat from competition that can significantly decrease or even more eliminate the option value and exercise its option at the optimum time compensating PTC and uncertainty control.

Figure 2. Waiting and Operation Period for a Single Real Option (ts=0)
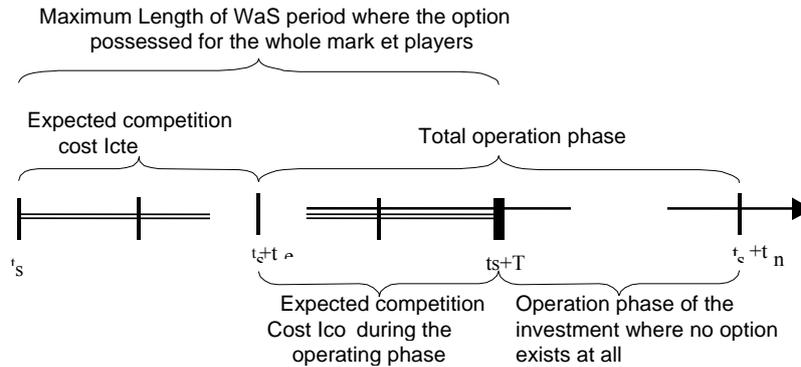
After the implementation of the investment (option exercise) the Firm may also experience PTC up to time T that can further decrease its expected value of the operation's revenues. The target of the Firm is to pre-empt the subsequent competitors, after this time. However, in case of hard competition, as it is in the ICT field where many competitors are sharing the same option, this is not realistic. Alternatively, the Firm wants to minimize the effect of competitors' arrivals during the operation phase. Hence, an important characteristic for each business opportunity is to provide a strong capability for the Firm to pre-empt subsequent competitors' entry after its entry in the market. At exercise time t_e, let I_{cwte} be the competitive erosion per year of competitors who have already enter into the market. Let also V be the overall market investment revenues when no competition exists at all. Then, the revenues of the investment opportunity, which are available to the Firm are $V - I_{cwte}*t_e$. This value is fully available to the Firm when there is full PCI to the following competitors, so no competitor arrival is expected during the operation phase. However, as mentioned before, it seems more realistic to consider that a number of subsequent competitors can also enter the market after Firm's entry into the market. We model a partial PIC by considering that during operation phase and up to t=T, competitors may also arrive and subtract part of the available to Firm investment value. The smaller this part is, the higher the PCF is. Hence, the final investment value that will be available to the Firm is given by:

$$V_f = V - I_{cwte}*t_e - I_{co}*(T-t_e) \qquad\qquad (3)$$

The magnitudes of Icwte and Ico depend on the competition intensity and the number of players, which are finally entering the market (Angelou and Economides 2006a, b).

In this work we model the yearly competition costs during waiting period and operation period as I_{cwte} and I_{co} respectively. Actually, we define as I_{cwte} the competition cost per year during WaS period, while I_{co} the competition cost per year during operation period. Hence the option value under competition threat during waiting and operation period for ts=0 is given by:

$$OV_{cte} = \max\left(V - I_{cwte}*t_e - I_{co}*(T - t_e) - X, 0\right) \qquad\qquad (4)$$

We consider that investment revenue V, competition costs parameters Icwte and Ico and one-time investment cost X are following a joint-diffusion process. In addition we adopt Fuzzy Logic analysis to model their expected present values at decision time.

Adopting triangular fuzzy logic numbers we define as

$$\text{V: } [V_L, V_M, V_H]$$
$$\text{X: } [X_L, X_M, X_H] \qquad\qquad (5)$$
$$\text{I}_{cwte}\text{: } [I_{cwteL}, I_{cwteM}, I_{cwteH}]$$
$$\text{I}_{co}\text{: } [I_{coL}, I_{coM}, I_{coH}]$$

The Option Value is given by

$$\text{OV}_{cte}\text{: } [OV_{cteL}, OV_{cteM}, OV_{cteH}] \qquad\qquad (6)$$

Hence, we have

$$OV_{cteL}= max(V_L-I_{cwteL}*t_e-I_{coL}*(T-t_e)-X_L,0)$$
$$OV_{cteM}= max(V_M-I_{cwteM}*t_e -I_{coM}*(T-t_e)-X_M,0) \qquad (7)$$
$$OV_{cteH}= max(V_H-I_{cwteH}*t_e -I_{coH}*(T-t_e)-X_H,0)$$

Angelou and Economides (2006a, b) analyze the cases for PCF as well as the correlation between V and competition costs. Especially, in case of "No PCF" it is more preferable to wait up to time T, since V_f will be the same independently of the option exercise strategy. In case of "Full PCF" there are two effects negatively correlated between each other: i) the uncertainty control assured by both the ROs analysis and the managerial flexibility to deploy investment in a longer deferral period and ii) the PTC that may fully eliminate the option value for the firm. Finally, in case of "Partial PCF" by investing earlier a level of preemption capability can be achieved. It might be optimal for the firm to invest earlier in order to ensure the highest possible level of the investment's revenues. Of course, it is still a matter of compensation between managerial flexibility and CT as before.

Incentive of investing earlier can also be applied when WaS strategy results to significant revenues losses from the operation phase that overcome the value of the uncertainty control provided by the ROs approach. A divided yield parameter may indicate these revenues losses (Trigeorgis 1996). Here, we assume that this divided yield is zero.

Competition cost can be either positively or negatively correlated with V. Someone may assume that the bad business conditions compared to the favorable ones experience no network externalities effects. Also, the bad business conditions indicate no achievement of the critical mass for the customers demand indicating so a relatively small subtraction of the overall investment opportunity available to the firm. The opposite can be in case of favorable business conditions. In addition, there can be cases, where while the market value appears appealing, the competitors cannot extract significant option value. Particularly, when competitors do not have the adequate ICT infrastructure to fully utilize their own investment's opportunity benefits, an increase of the overall market value

V might finally decrease the part of the market share that a specific competitor can subtract from firm.

In another point of view smaller correlation values can be applied in real life cases under competitors' asymmetries such us investment cost, initial infrastructure ownership and other physical resources availability. Especially, Firm owns a competitive advantage against the rest of competitors coming from the physical resources availability.

In addition, concerning correlation between investment revenue V and one time cost X a negative value could represent, for instance, that the inability to control the costs of the development project are associated with lower revenues after the project/phase is completed.

We consider, a joint diffusion process for the I_{cwte}, I_{co}, V and X, Figure 3 in Appendix A. We adopt an extended log transformed binomial model (ELTBM) with 4-parameters that follow joint diffusion process (Gamba and Trigeorgis 2001). For small number of steps or volatilities values of the stochastic parameters with respect to the rf, the Binomial Method becomes unstable since the up and down probabilities of asset parameters can be negative. ELTBM does not present this disadvantage being so fully stable and efficient.

A REAL-WORLD CASE STUDY

In this work the firm under investigation is Egnatia Odos S.A. (EO). Its core business activity is the management of design and construction, the operation, maintenance, and exploitation of the 680 kilometres long "Egnatia Odos" motorway (EOM). The transportation network of EO can be used for the installation of optical network backbone infrastructure along it. The commercial exploitation of this network is the business activity to be analysed. First, EO decides to enter the market of broadband networks, installing optical dark fibers, along the EOM, looking afterwards for their commercial exploitation. Second, EO goes a step ahead and decides to light the optical fibers. This means that the customers are able to buy wavelengths. Hiring wavelengths requires the installation, operation management and maintenance of active equipment. We consider this opportunity as a growth real option.

We assume that EO has proceeded in the implementation on phase 1, installation of optical network backbone infrastructure. The growth option value stemmed from EO's

belief that they could resolve some of the uncertainties. Such uncertainties are the broadband services demand in the region of interest and the capability of the company to enter a new market. EO may adopt the strategy to wait and learn more about the investment, to be able to better assess it and subsequently avoid it if the expected revenues turned out to be unattractive. However waiting to learn more is not without cost. Actually, since the specific growth ROs are shared among competitors in the market they may experience significant degradation by first movers (competitors) entry into the market. Our target is to consider the optimum time to implement the second phase investment, lighting the fibre. Iatropoulos *et al.* (2004) provide analysis for the market demand for the EO business case. We assume that the EO as well as the rest of the competitors posse a shared RO that can be exercised up to t=T. The results of our analysis show that sometimes the EO may be better to adopt longer WaS period despite of the PTC that may eliminate the option value.

For the estimation of the optimum deployment strategy of the investment we follow the rule suggested by Benaroch and Kaufman (2000) and applied by Iatropoulos *et al.* (2004).

• **Decision Rule**: Where the maximum deferral time is T, make the investment (exercise the option) at time te, $0 < t_e < T$, for which the option, OV_{cte}, takes on its maximum value.

$$OV_{cte} = \max(t=0\ldots T)\ OV_{ct} \tag{8}$$

We do not provide real numbers, to protect EO confidentiality, but present the application of our model to a real life case study. However, the whole analysis is based on discussions and interviews, which we had, with the ICT management of the company.

Next, we present the results of our analysis for three exercise times, $t_e = 1, 2, 3$ Table 3.We model partial PCI assuming that I_{co} is smaller than I_{cwte}.

Table 3. Fuzzy Option Value under Competition Threat for t_e=1, 2, 3 (EgnatiaOdos S.A. Case Study)

	óv	40%	óx	20%	$\tilde{n}_{V,X}$	0		

		$\tilde{n}_{V,I}$		0				
Ó Iw, Ico		te=1			te=2			te=3	
	OV_L	OV_M	OV_H	OV_L	OV_M	OV_H	OV_L	OV_M	OV_H
0%	4,675	5,79	6,9	7,43	9,22	12,08	8,86	12,43	16,638
40%	4,673	6,04	7,7	8	11,27	14,53	11,14	15,86	20,486
80%	4,659	7,23	9,8	8,935	13,28	17,75	14,2	21,42	28,64

		$\tilde{n}_{V,I}$		0,8				
Ó Iw, Ico		te=1			te=2			te=3	
	OV_L	OV_M	OV_H	OV_L	OV_M	OV_H	OV_L	OV_M	OV_H
0%	4,675	5,79	6,9	7,43	9,22	12,08	8,86	12,43	16,638
40%	3,114	2,98	2,761	4,69	5,14	5,59	5,84	7	8,2
80%	1,23	1,43	1,63	2,38	3,37	4,5	5,5	8,8	12,11

V_L	V_M	V_H
80	100	120
X_L	X_M	X_H
90	100	110
Iwte_L	Iwte_M	Iwte_H
10	15	20
Ico_L	Ico_M	Ico_H
5	10	15

As it can be seen, the longer WaS period may indicate higher option values, for the specific values of competition parameters taken here, despite PTC to eliminate part of the investment value. In general as mentioned before, it is a matter of compensation between, uncertainty control assured by ROs thinking and competition threat caused by the incoming competitors during WaS and operation period for the Firm. In our example, we consider that the maximum length of WaS period is three years.

The specific investment is marginally "out of the money." In this case ROs analysis provides higher performance value for the investment opportunity treated as RO.

If RO is "deep in money" meaning that NPV value is clearly positive WaS strategy may be less optimum (see Table 4).

Table 4. Fuzzy Option Value under Competition Treat for t_e=1, 2, 3 (when NPV is clearly positive)

σ_V 30% σ_X 20% $\tilde{n}_{V,X}$ 0

$\tilde{n}_{V,I}$ 0,8

$\sigma_{Iw,Ico}$	te=1			te=2			te=3		
	OV_L	OV_M	OV_H	OV_L	OV_M	OV_H	OV_L	OV_M	OV_H
0%	14,8	17,3	19,84	16,6	18,41	20,16	16,7	17,34	17,68
40%	11,7	11,1	10,48	11,3	7,67	5,15	9,3	5,26	5,35
80%	7,7	6	6,06	6,9	9,22	11,46	11,1	16,1	21,14

V_L	V_M	V_H
85	120	155
X_L	X_M	X_H
60	80	100
$Iwte_L$	$Iwte_M$	$Iwte_H$
10	20	30
Ico_L	Ico_M	Ico_H
5	10	15

DISCUSSION AND FUTURE RESEARCH

Our method is considered as extention of Benaroch (2000) and Iatropoulos *et al.* (2004) work for finding the optimum deployment strategy for investment opportunities in information technology and broadband technology fields respectively. In particular, we introduce competition modelling to estimate the threat of competitors that the firm of interest is experiencing during waiting period before investing. We also propose fuzzy logic analysis to model imprecision in expected values of our model parameters.

A limitation of our model can be in the way we estimate the up and down coefficients in the joint diffusion process of competition, investment costs and investment revenues. We adopt the risk neutral probabilities for competition parameters in a similar way as the overall market value V. These assumptions may be an issue of criticism that requires further discussion for their validation. However, our intention is to show how the uncertainty in competition parameters influences the value of a future investment opportunity being treated as RO.

In our analysis we consider one time step joint-diffusion process. Of course, multiple time steps result to increased granularity and so to increased accuracy in the results. Though the complexity of the model is increasing dramatically we capture more efficiently the additional dimension of competition entry.

Finally, our model is more valid for high number of competitors in the market where the movement of each of them does not influence the overall market status. In case of

oligopoly market conditions, where the movement of one competitor causes the answer of another competitor, the endogenous competition modelling is more efficient. In particular, someone could adopt endogenous competition modelling assuming that each one of the competitors in the market experiences a different level of the competition cost parameters. Actually, the smaller values these parameters for a player in the market are, the stronger its market position for the specific investment opportunity is. In this case endogenous competition modelling requires the integration of ROs with Game Theory.

CONCLUSIONS

Despite its appearance, the fuzzy real options model is quite practical and useful. The imprecision we encounter when judging or estimating future cash flows is not only stochastic in nature since the uncertainty may be genuine, i.e. we simply do not know the exact levels of future cash flows. Without introducing fuzzy real option models it would not be possible to formulate this genuine uncertainty. The proposed model that incorporates subjective judgments and ROs analysis may give investors a better understanding of the problem when making investment decisions.

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APPENDIX A

Figure 3. Investment Revenue V and Cost X, Competition Costs I_{cwte}, I_{co} Joint Diffusion Process during WaS and Operation Period (one time step)
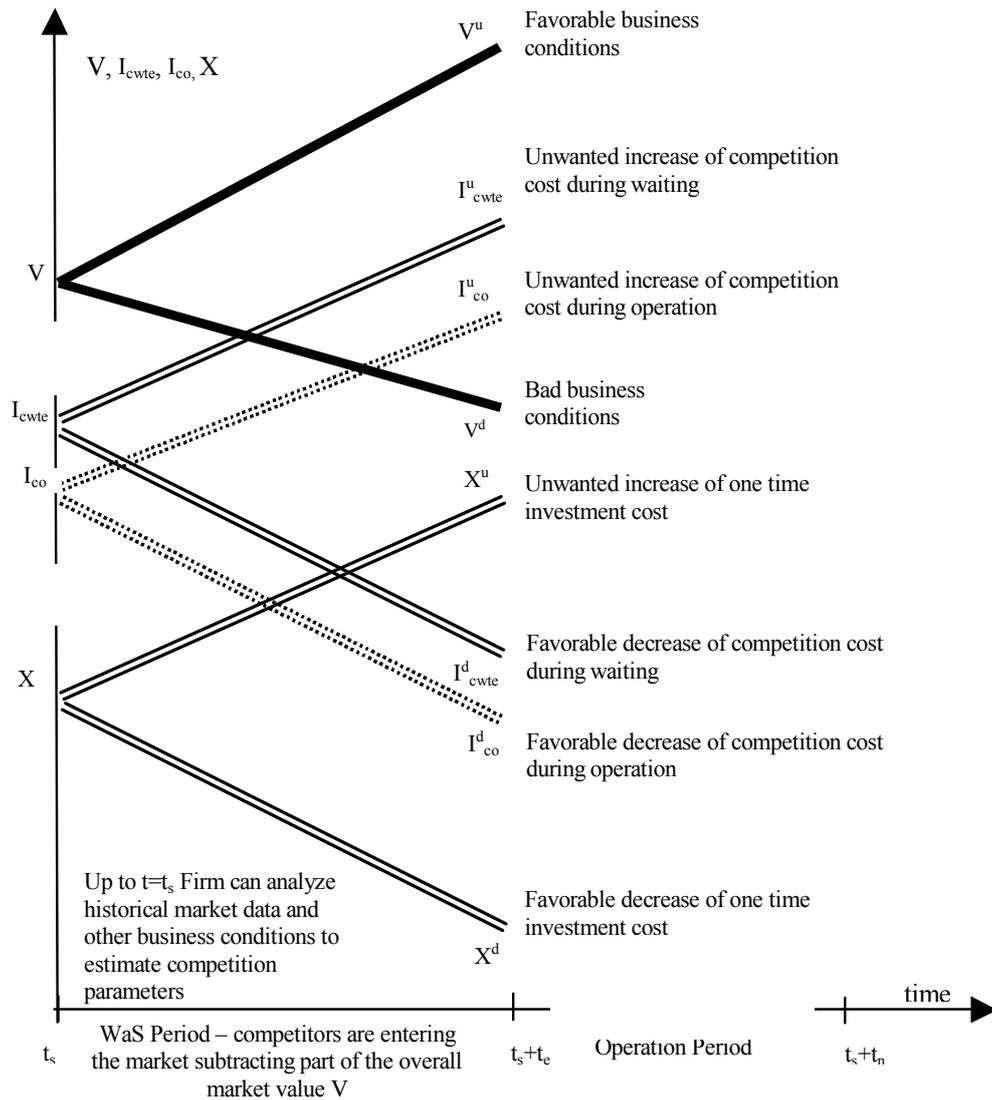
APPENDIX B

Table 2. Notations Used in the Proposed Mathematical Model

Parameter	Description
t_s	Time where the option is possessed for the first time by the Firm and the rest of competitors.
T	Maximum deferral period in years for the option to be exercised at t_s+T. We assume that T is the same for all the competitors in the market.
t_e	Time where the option is finally exercised by the Firm and the investment is implemented. Final waiting period is t_e-t_s.
$[V_L, V_M, V_H,]$	The overall market value for the growth investment opportunity.
FOV_{cte}	Fuzzy Option value under exogenous competition modeling when it is exercised at $t=t_s+t_e$.
$[I_{cwTL}, I_{cwTM}, I_{cwTH}]$	Total competitive erosion during waiting period up to t_s+T
$[I_{cwteL}, I_{cwteM}, I_{cwteH}]$	Total competitive erosion during waiting period up to t_e, where $t_s<t_e<t_s+T$
$[I_{coL}, I_{coM}, I_{coH}]$	Total competitive erosion during operation period after option exercise at $t=t_e$.
I_c	$I_{cwte} +I_{co}$, total competitive erosion cost.
V_f	$V-I_c$. Final investment revenues for the Firm.
R	The risk free interest rate.
$[X_L, X_M, X_H]$	Investment One-time cost.
σ_v	Investment revenues uncertainty V.
σ_{Icwte}	Competition cost during WaS period I_{cwte} uncertainty (volatility).
σ_{Ico}	Competition cost during operation I_{co} uncertainty (volatility).